Available online at www.sciencedirect.com
ScienceDirect
International Journal of Multiphase Flow 32 (2006) 1294–1299
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Brief communication

The collapse of gas bubbles and cavities in a viscoelastic fluid

J. Jiménez-Fernández *, A. Crespo

Dpto. Ingeniería Energética y Fluidomecánica, E.T.S. Ingenieros Industriales, UPM, 28006 Madrid, Spain

Received 7 July 2005; received in revised form 3 June 2006

1. Introduction

A great deal of attention has been devoted to study the growth and collapse of bubbles in viscoelastic media. The interest has been motivated by experimental works where it has been shown that cavitation may be reduced or even suppressed in polymeric water solutions (see Brujan et al. (2004) and references therein). Correspondingly, numerous theoretical analyses have been devoted to study the collapse of spherical bubbles in viscoelastic fluids, induced by an imposed pressure difference between the internal gas pressure, and the external pressure of the continuous phase. Revisions of the empirical as well as the theoretical works on the subject may be found in the book of Levitskiy and Shulman (1995) and in Brujan (1999), where the problem is analyzed taking into account the effects of the liquid compressibility. Pioneer theoretical studies on voids Fogler and Goddard (1970), as well as on gas bubbles Tanasawa and Yang (1970), showed that oscillatory behavior is enhanced by the elasticity of the fluid, so that in non-Newtonian media, it must be expected that the collapse occurs by a sequence of successive rebounds damped by viscosity. Zana and Leal (1975) investigated the diffusion-induced collapse of gas bubbles in viscoelastic liquids, and found significant differences between the Newtonian case and the viscoelastic case for both the collapse rate and the internal bubble pressure. Analogous results have been obtained in subsequent works, (Kim, 1994; Brujan, 1999), where it has been also claimed that in high elastic non-Newtonian fluids, the collapse process becomes close to the one corresponding to an inviscid fluid. Nevertheless, these theoretical predictions have been not fully confirmed in experiments. On the contrary, it should be concluded from empirical works, that the dynamics of single bubbles is unaffected by the rheological properties of the host fluid, except for in the proximity of rigid boundaries (Chahine and Fruman, 1979; Brujan et al., 1996, 2004), or in the case of non-spherical bubbles, Hara and Schowalter (1984). Furthermore, it has been also shown that the bubble dynamics experimentally observed in shear thinning polymeric solutions is more suitably described ignoring the non-Newtonian effects, and considering the infinity-shear viscosity as the viscosity of the polymer solution Brujan et al. (1996). Therefore, it is not clear at present if, in an unbounded fluid, the rheological nature of the host fluid has or not, an explicit influence on the individual bubble behavior.

In this note, the collapse of a spherical gas bubble as well as a spherical empty cavity in a viscoelastic liquid are revised by considering for the extra stress tensor a differential constitutive equation with an interpolated time derivative. This rheological model here adopted is adequate to the study of bubble dynamics because, as shown by Shulman and Levitskiy (1987), Levitskiy and Shulman (1995) the generalized Rayleigh–Plesset

* Corresponding author. Tel.: +34 91 3364156; fax: +34 91 3363006.
E-mail address: jajimenez@enerflu.etsii.upm.es (J. Jiménez-Fernández).

0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2006.06.001

equation which is usually in the viscoelastic case an integrodifferential equation, may be reduced to an ordinary differential equation. This procedure has been used by Gibaudullin et al. (2001) to study shock waves in non-Newtonian bubbly liquids and more recently by Jiménez-Fernández and Crespo (2005) to analyze acoustically driven oscillations of bubbles in viscoelastic fluids. The full governing system has been numerically solved and the differences in behavior between a gas bubble and an empty cavity are discussed. It is shown that under the action of elastic normal stresses, gas bubbles with small gas content and empty cavities may have a similar behavior in the early phases of the collapse.

Previous theoretical predictions have been confirmed. Furthermore, it has been found that as the fluid elasticity of a polymeric solution is increased, the elastic effects and the viscous effects due to polymer contribution cancel out, so that the collapse process in the polymeric solution becomes close to the collapse process in a Newtonian fluid, with a viscosity equal to the viscosity of the Newtonian solvent.

Thus, this opposite and simultaneous action of fluid elasticity and viscous damping may provide a first explanation about the small differences experimentally observed between the bubble collapse processes in Newtonian fluids and viscoelastic fluids.

2. Formulation

Consider a spherical gas-vapor bubble of radius R immersed in a viscoelastic fluid of density ρ. The dynamics of the bubble is governed by the Rayleigh–Plesset equation which may be written in the form:

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=p_{\mathrm{g}}-p_{\infty}+p_{\mathrm{v}}-\frac{2\sigma}{R}+2\int_R^{\infty}\frac{\tau_{\mathrm{rr}}-\tau_{\theta\theta}}{r}\,\mathrm{d}r \tag{1}$$

where the dot denotes time differentiation, p_{g} is the gas bubble pressure, p_{∞} the pressure of the liquid far away from the bubble, p_v the vapor pressure, σ the coefficient of surface tension and τ_{rr}, $\tau_{\theta\theta}$ the components of the extra stress tensor $\boldsymbol{\tau}$. In the following it will be assumed that the extra stress tensor $\boldsymbol{\tau}$ is given by a constitutive equation of differential type in the form:

$$\boldsymbol{\tau}=\boldsymbol{\tau}^p+\boldsymbol{\tau}^s \tag{2}$$

where

$$\boldsymbol{\tau}^s=2\eta_s\mathbf{e} \tag{3}$$

is the contribution to the stress of the Newtonian solvent and $\boldsymbol{\tau}^p$ is the polymer contribution which satisfies the equation:

$$\boldsymbol{\tau}^p+\lambda\left(\frac{D\boldsymbol{\tau}^p}{Dt}-a(\boldsymbol{\tau}^p\mathbf{e}+\mathbf{e}\boldsymbol{\tau}^p)\right)=2\eta_p\mathbf{e} \tag{4}$$

In the above expressions, η_s is the solvent viscosity, $\mathbf{e}$ is the rate of strain tensor defined as: $\mathbf{e}=\frac{1}{2}(\nabla\mathbf{v}+(\nabla\mathbf{v})^{\mathrm{T}})$ where $\nabla\mathbf{v}$ is the velocity gradient, $\frac{D\tau^p}{Dt}=\frac{\partial\tau^p}{\partial t}+\mathbf{v}\nabla\tau^p$ is the convective derivative, λ is the stress relaxation time and η_p is the polymer contribution to the shear viscosity of the solution: $\eta=\eta_s+\eta_p$. Eq. (4) may be also written in an alternative form by introducing a retardation time defined as $\lambda'=\lambda(\eta_s/\eta)$. The term in large brackets in expression (4) is an invariant derivative for any value of a in the interval: $-1\leqslant a\leqslant 1$ Joseph (1990). For $a=1$, the Eq. (4) is the Oldroyd-B model, a rheological model which predicts a constant shear viscosity and strain thickening of the extensional viscosity. For intermediate values of a in the interval $0\leqslant a<1$, Eq. (4) leads to models which exhibit a shear thinning viscosity. Different constitutive equations, defined by different values of the parameter a have been used in the past for this problem. The case $a=0$ was considered by Tanasawa and Yang (1970), and by Brujan (1999), the case $a=1$ by Ting (1975) and finally the case $a=1$ and $\eta_s=0$, (Upper Convected Maxwell model) by Kim (1994). All these models, convert the Rayleigh Plesset Eq. (1) into an integrodifferential equation, after a transformation to Lagrangian coordinates. However, for $a=1$ and $a=1/2$, Eqs. (1)–(4) may be reduced to the following differential system (Levitskiy and Shulman, 1995; Jiménez-Fernández and Crespo, 2005):

$$\rho\left(R\ddot{R}+\frac{3}{2}\dot{R}^2\right)=p_g-p_\infty+p_v-\frac{2\sigma}{R}-4\eta_s\frac{\dot{R}}{R}+S_p^{(1)}+(2a-1)S_p^{(2)} \tag{5}$$

$$\dot{S}_p^{(1)}=-\left(\frac{1}{\lambda}+4a\frac{\dot{R}}{R}\right)S_p^{(1)}-\frac{2}{a}\frac{\eta_p}{\lambda}\frac{\dot{R}}{R} \tag{6}$$

$$\dot{S}_p^{(2)}=-\left(\frac{1}{\lambda}+\frac{\dot{R}}{R}\right)S_p^{(2)}-2\frac{\eta_p}{\lambda}\frac{\dot{R}}{R};\quad a=1\text{ or }a=\frac{1}{2} \tag{7}$$

where $S_p(t)=S_p^{(1)}+(2a-1)S_p^{(2)}$ corresponds to the integral on the normal stresses difference in (1). Thus, these particular values of the parameter a lead to a more simpler governing initial value problem which may be integrated by means of standard numerical methods. Computation times are therefore reduced and convergence difficulties in the neighbourhood of the collapse point previously reported: Ting (1975), Kim (1994), Brujan (1999), may be also avoided.

In the following, it will be assumed that the surface tension as well as the vapor pressure are negligible. It will be also considered that the gas bubble pressure is given by a polytropic transformation: $p_g=p_0\,(R_0/R)^{3\gamma}$, where p_0 and R_0 are respectively, the initial gas bubble pressure and the initial bubble radius.

The above equations are written in dimensionless form by means of the following scales: length: R_0, time: $t_c=R_0/\sqrt{\frac{p_\infty}{\rho}}$, and pressure: $p_c=p_\infty$. With these scales, Eqs. (5)–(7) become

$$R^*\ddot{R}^*+\frac{3}{2}\dot{R}^{*2}+\frac{4\varepsilon}{Re}\frac{\dot{R}^*}{R^*}-\frac{q}{R^{*3\gamma}}+1=S_p^{*(1)}+(2a-1)S_p^{*(2)} \tag{8}$$

$$\dot{S}_p^{*(1)}+\left(\frac{1}{De}+4a\frac{\dot{R}^*}{R^*}\right)S_p^{*(1)}=-\frac{2(1-\varepsilon)}{aDe\,Re}\frac{\dot{R}^*}{R^*} \tag{9}$$

$$\dot{S}_p^{*(2)}+\left(\frac{1}{De}+\frac{\dot{R}^*}{R^*}\right)S_p^{*(2)}=-\frac{2(1-\varepsilon)}{De\,Re}\frac{\dot{R}^*}{R^*},\quad a=\frac{1}{2},1 \tag{10}$$

where the star (which will be suppressed hereafter) denotes dimensionless quantities. $Re=\rho R_0^2/t_c\eta$, is the Reynolds number, $De=\lambda/t_c$ is the Deborah number, $\varepsilon=\eta_s/\eta$ and $q=p_0/p_\infty$.

3. Results and conclusions

System (8)–(10) has been integrated with the initial conditions: $R(0)=1,\dot{R}(0)=S_p^{(1)}(0)=S_p^{(2)}(0)=0$. So, it is implicitly assumed in the model that the bubble, at rest at the pressure p_0 for times prior to zero, is out from equilibrium at the initial instant when the external pressure is suddenly increased to p_∞. When $q=0$, (empty cavity) the strain rate $\dot{R}/R$ and correspondingly the normal stresses S_p tend to constant values for large times, so that the bubble radius tends asymptotically to zero. On the contrary, if $q>0$, (gas bubble) the normal stresses S_p tend to zero, so that the bubble travels towards a new equilibrium state with bubble radius $R=R_f=q^{1/3\gamma}$. For large times, there is therefore a different behavior for the two cases $q=0$ and $q>0$. However, in the early phase of the collapse the numerical results obtained show a very similar behavior in the cases $q=0$ and $q>0$ for an Olroyd-B model when q is small, as is illustrated in the Fig. 1. In this figure, the influence of the pressure ratio q on the amplitude of the bubble radius at the first rebound, for the system: $Re=10$, $De=1$, $\varepsilon=0.2$, is shown. Note that, quantitative differences are found for an interpolated model whereas for an Oldroyd-B model a very similar behavior is indeed observed between a void and a gas bubble if $q<0.001$. In fact, in the early phases of the collapse the elastic normal stresses dominate over the compressive stresses generated by the gas inside the bubble as is illustrated in Fig. 2. In this figure, the difference of normal stresses S_p for both, a gas bubble ($q=10^{-4}$) and an empty cavity ($q=0$) as well as the internal bubble pressure $q/R(t)^{3\gamma}$ versus time, are plotted for an Oldroyd-B fluid with $De=0.5$, $Re=2$, and $\varepsilon=0.2$. As it may be observed, for times prior to $t\approx 3$, the difference of normal stresses S_p takes the same values for $q=0$ and $q=10^{-4}$, whereas the gas bubble pressure is nearly zero. Consequently, the corresponding curves for $R(t)$, which have been also plotted, are in both cases identical in this phase of the collapse. For larger times the gas bubble has reduced its radius sufficiently for the internal pressure begin to take significant values so that the ratio $q/R^{3\gamma}(t)$ grows to unity, as the bubble radius tends to the equilibrium value R_f. Simultaneously, S_p tends to zero. By contrast, in an empty

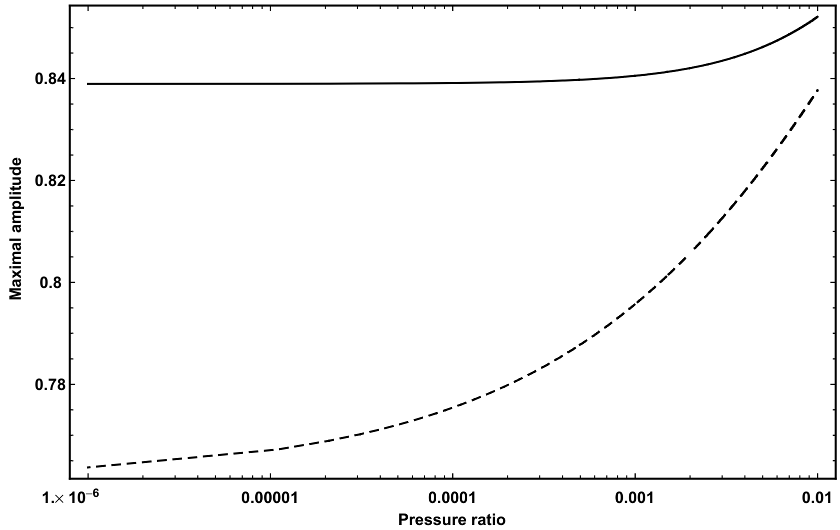

Fig. 1. Bubble radius at the first rebound versus the pressure ratio $q = p_0/p_\infty$ for an interpolated model (dashed line) and an Oldroyd-B fluid (solid line). $Re = 10$, $De = 1$, $\varepsilon = 0.2$.

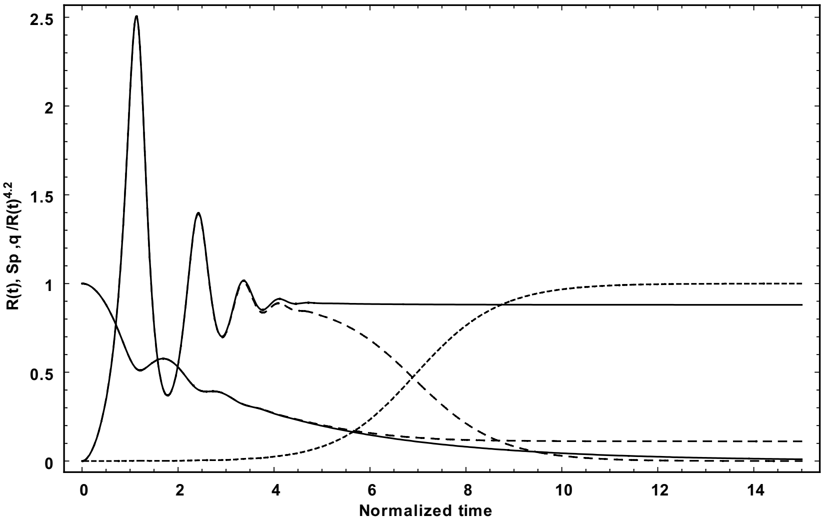

Fig. 2. The integrated difference of normal stresses S_p and the bubble radii versus time for an empty cavity (solid lines) and a gas bubble with $q = 10^{-4}$ (dashed lines) in a Olroyd-B fluid: $De = 0.5$, $Re = 2$, $\varepsilon = 0.2$. The dotted line is the dimensionless gas bubble pressure: $q/R(t)^{3\gamma}$.

cavity S_p tends to a constant value as the bubble radius tends to zero. This theoretical result, supports previous predictions on the collapse of empty cavities, namely, that voids which collapse catastrophically in Newtonian fluids, experience an oscillatory motion in viscoelastic fluids.

The net influence of the fluid elasticity, which is here quantified by the Deborah number, is shown in Fig. 3, where the maximal amplitude (that is the amplitude of the first rebound) as function of De is plotted for fixed $Re = 10$, $q = 10^{-4}, \varepsilon = 0, 0.2, 0.5$ and $a = 1/2$, 1 respectively. As it may be noted, this amplitude tends to zero as De tends to zero. Therefore, it is confirmed that for these values of Re and q, the collapse will be monotonous in a Newtonian fluid and oscillatory in a viscoelastic fluid. As expected, the amplitude increases with De but tends towards a constant value for moderate to large values of this parameter. Thus, if ε is very small or if ϵ vanishes (models like Maxwell) an inviscid behavior is observed. This is a result predicted in previous

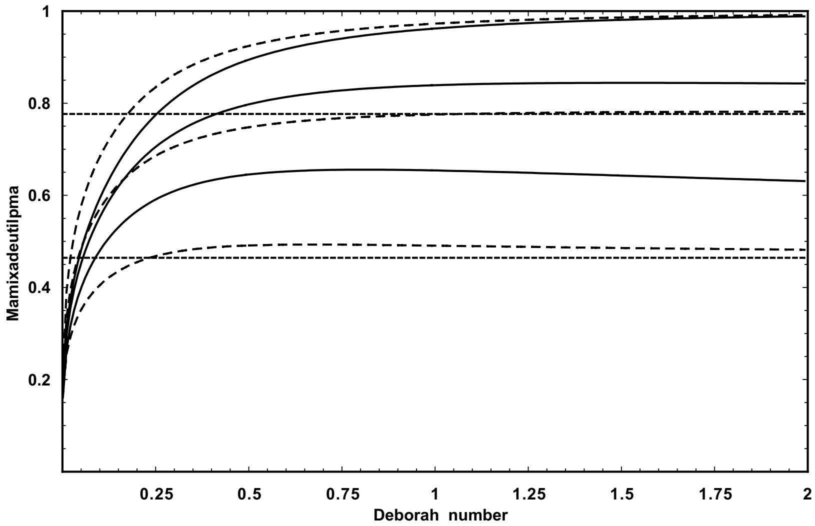

Fig. 3. Amplitude of the first rebound versus Deborah number for $q = 10^{-4}$, $Re = 10$ and $\varepsilon = 0, 0.2, 0.5$ from top to bottom, respectively. Dashed lines ($a = 1$). Dark lines ($a = 1/2$). Horizontal lines show the value of the amplitude in the Newtonian solvent.

works (Kim, 1994; Brujan, 1999) which should be considered as a theoretical prediction characteristic of models of Maxwell type. For non-vanishing values of ε, the amplitudes tend towards asymptotic values which are those corresponding to a Newtonian fluid if the Reynolds number is increased from $Re = 10$ to $Re_s = 10/\varepsilon$. These asymptotic behaviors correspond to the solutions of the system (8)–(10) for $S_p^{(1)} = S_p^{(2)} = 0$, i. e., the limiting values of $S_p^{(1)}$ and $S_p^{(1)}$ for $De \gg 1$, when Re is the order of the unity. For arbitrary Re, the asymptotic behavior for $S_p(t) = S_p^{(1)} + (2a-1)S_p^{(2)}$ for large De, may be determined from the Eqs. (9) and (10) and is given by the following expression:

$$S_p = -\frac{2(1-\varepsilon)}{De\,Re}\left(\frac{3+2a}{4} - \frac{7-6a}{4R^{4a}} - \frac{2a-1}{R}\right), \quad a = \frac{1}{2}, 1 \tag{11}$$

Thus, if the Reynolds number is the order of the unity or larger, as the Deborah number is increased, the influence of the polymeric normal stresses difference S_p is reduced, so that the collapse process in the polymeric solution is controlled by the viscosity of the Newtonian solvent, which becomes the unique rheological parameter involved. It must be remarked that, according to the rheological model considered in the present analysis for shear-thinning fluids ($a = 1/2$), the viscosity of the solvent is just the infinity-shear viscosity of the solution. This theoretical trend is therefore in good qualitative agreement with the experimental observations of Brujan et al. (1996) for aqueous polymeric solutions.

Acknowledgments

The authors wish to acknowledge the financial support of the Dirección General de Investigación. Ministerio de Ciencia y Tecnología (Spain). Project: DPI2003-05643. Numerical calculations have been performed with Mathematica® (Wolfram Research Inc.).

References

Brujan, E.A., 1999. A first-order model for bubble dynamics in a compressible viscoelastic liquid. J. Non-Newtonian Fluid Mech. 84, 83–103.

Brujan, E.A., Ikeda, T., Matsumoto, Y., 2004. Dynamics of ultrasound-induced cavitation bubbles in non-Newtonian liquids and near a rigid boundary. Phys. Fluids 16, 2402–2410.

Brujan, E.A., Ohl, C.D., Lauterborn, W., Philipp, A., 1996. Dynamics of laser-induced cavitation bubbles in polymer solutions. Acoustica 82, 423–430.

Chahine, G.L., Fruman, D., 1979. Dilute polymer solution effects on bubble growth and collapse. Phys. Fluids 22, 1406–1407.
Fogler, H.S., Goddard, J.D., 1970. Collapse of spherical cavities in viscoelastic fluids. Phys. Fluids 13, 1135–1141.
Gibaudullin, A.A., Beregova, O.Sh., Bekished, S.A., 2001. Shock waves in non-Newtonian bubbly liquids. Int. J. Multiphase Flow 27, 635–655.
Hara, S.K., Schowalter, W.R., 1984. Dynamics of non spherical bubbles surrounded by viscoelastic fluid. J. Non-Newtonian Fluid Mech. 14, 249–264.
Jiménez-Fernández, J., Crespo, A., 2005. Bubble oscillation and inertial cavitation in viscoelastic fluids. Ultrasonics 43, 643–651.
Joseph, D.D., 1990. Fluid Dynamics of Viscoelastic Liquids. Springer-Verlag, New York Inc.
Kim, Ch., 1994. Collapse of spherical bubbles in Maxwell fluids. J. Non-Newtonian Fluid Mech. 55, 37–58.
Levitskiy, S.P., Shulman, Z.P., 1995. Bubbles in Polymeric Liquids: Dynamics, Heat and Mass Transfer. Technomic Publishing A G, Bassel, Switzerland.
Shulman, Z.P., Levitskiy, S.P., 1987. Closure of a cavity in polymeric liquids. J. Eng. Phys. 53, 893–897.
Tanasawa, I., Yang, W.-J., 1970. Dynamic behavior of a gas bubble in viscoelastic liquids. J. Appl. Phys. 41, 4526–4531.
Ting, R.Y., 1975. Viscoelastic effect of polymers on single bubble dynamics. AIChE J. 21, 810–813.
Zana, E., Leal, L.G., 1975. Dissolution of a stationary gas bubble in a quiescent viscoelastic liquid. Ind. Eng. Fundam. 14, 175–182.